Spectra of Ti^{51}, Cr^{53}, and Fe^{55} and the Unified Model*

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The spectrum and level properties of Ti^{51} have been investigated by the reaction $Ti^{50}(d,p)Ti^{51}$ using a high-resolution magnetic analysis system. The four nuclei Ca^{49}, Ti^{51}, Cr^{53}, and Fe^{55} have an odd neutron outside a closed $f_{7/2}$ neutron shell. The spectrum of Ca^{49} as observed in a (d,p) reaction on Ca^{48} is single-particle-like. In the other nuclei, the neutron single-particle states are fragmented to varying degrees. An attempt has been made to study the information available for Ti^{51}, Cr^{53}, and Fe^{55} on the basis of the unified model by coupling the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ shell model states to a quadrupole vibrational core. Core states of up to three phonons have been included in the calculation. Reasonable agreement has been obtained with the excitations and transition strengths of the low-lying levels of the three nuclei as observed in (d,p) reactions on the corresponding even-even nuclei. In the case of Cr^{53} which is stable, additional experimental information is available, viz., the ground state magnetic moment, the static quadrupole moment and the lifetime of the 2.32-MeV level. Using the model wavefunctions obtained for Cr^{53}, these quantities have been computed and found to be in poor agreement with the experimentally observed values.

I. INTRODUCTION

FOR a systematic study of level structure of nuclei in the $1f-2p$ shell, the nuclei Ca^{49}, Ti^{51}, Cr^{53}, and Fe^{55} are particularly suitable on account of the following simplifying features: (1) These four nuclei have a single neutron outside a closed $f_{7/2}$ neutron shell and (2) these nuclei can be reached by the (d,p) reaction on the corresponding even-even stable nuclei. If core excitations were not important, the spectrum observed in such cases should be single-particle-like. However, considerable fragmentation of the single particle states was observed in the case of Cr^{53} and Fe^{55} [1–4] in contrast to the situation in Ca^{49}.[5]

It was decided to investigate the spectrum and level properties of Ti^{51} via the $Ti^{50}(d,p)Ti^{51}$ reaction since little information[6] was available in this case. The experimental procedure and results are presented in Sec. II, a preliminary account of which was given earlier.[7]

It was observed that the level structure of Ti^{51} shows similarities with that of Cr^{53} and Fe^{55} and is indicative of fragmentation of single particle states. To understand this fragmentation, a calculation based on the "unified model" is made and the results of the calculation for Ti^{51}, Cr^{53}, and Fe^{55} are discussed in Sec. III.

II. EXPERIMENTAL PROCEDURE AND RESULTS

Targets used in the present experiment were prepared by evaporating titanium dioxide enriched in Ti^{50} onto thin gold-leaf backing.[8] The isotopic composition of the target was Ti^{46} (1.69%), Ti^{47} (1.45%), Ti^{48} (10.5%), Ti^{49} (3.03%), and Ti^{50} (83.3%). The targets were exposed to the 7.8-MeV deuteron beam of The University of Michigan 42-in. cyclotron. Protons resulting from the (d,p) reaction on the target nuclei were analyzed by a high-resolution magnetic analysis system[9] with an over-all resolution of 30 keV, and were detected on photographic emulsions.

A typical proton spectrum covering a range of about 4.5 MeV is shown in Fig. 1. The various titanium groups were identified by their kinematic shift with angle. Due to the isotopic composition of the targets, proton groups corresponding to excitations of Ti^{51} and Ti^{49} were observed, apart from the ever present groups due to carbon and oxygen contamination of the target. The more intense groups of Ti^{49} could be picked out by comparing the spectrum obtained in this experiment with that obtained with an enriched Ti^{48} target.[10] An excitation of 3 MeV in Ti^{51} corresponds to about 4.8 MeV of excitation in Ti^{49} as determined from the ground-state Q values for the two reactions.[11,12] The spectrum of Ti^{49} at this excitation is quite rich and in the present experiment provided an unresolved background. Between 3 and 4.6 MeV excitation in Ti^{51}, five proton groups corresponding to levels of Ti^{51} were observed. No attempt was made to identify the weaker levels of Ti^{51} in this region. Angular distributions of the prominent groups of Ti^{51} were obtained at twelve different angles over an angular range of 10° to 90°; four typical angular distributions with neutron capture angular momentum $l_n=1$, 2, 3, and 4 are shown in

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[12] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.).

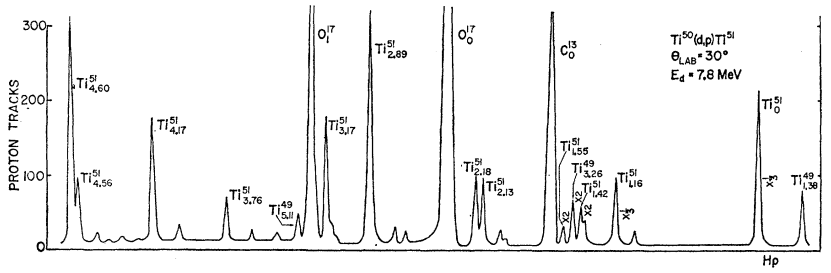

FIG. 1. Spectrum of Ti^{51} at 30°, as obtained from the reaction $Ti^{50}(d,p)Ti^{51}$ using a TiO_2 target (83% enriched in Ti^{50}).

Fig. 2. Values of l_n and relative reduced widths were obtained using Butler theory in those cases for which the experimental angular distributions could be fitted reasonably with Butler curves.

The excitation energies, neutron angular momentum transfer (l_n) values and the relative reduced widths obtained for Ti^{51} are summarized in Table I. The excitations are believed to be accurate to ±20 keV and the relative cross sections to ±10%. A level at 0.61 MeV reported previously[6] has not been observed in the present experiment. Four levels at 0, 1.16, 2.18, and 2.89-MeV excitation were observed to have $l_n=1$. On account of their low intensities (approximately 1/50th of the ground-state intensity) the intensities of levels at 1.42 and 1.55 MeV could be obtained only at five angles from 25° to 45°; no significant variation in intensity could be detected over this angular range. A level at 2.13 MeV shows an angular distribution typical of $l=3$ while a level at 3.76 MeV can be fitted reasonably with an $l=4$ Butler curve. Two levels at 4.17 and 4.60 MeV have angular distributions which can be fitted with $l=2$ Butler curves. For levels at 3.17 and 4.56 MeV, the complete angular distributions could not be obtained due to interference from C^{13} and O^{17} groups and hence l_n values could not be assigned to these levels. The levels and their known l_n values of Ti^{51} are plotted in Fig. 3, in which similar information relating to Ca^{49}, Cr^{53}, and Fe^{55} is included for purposes of comparison.

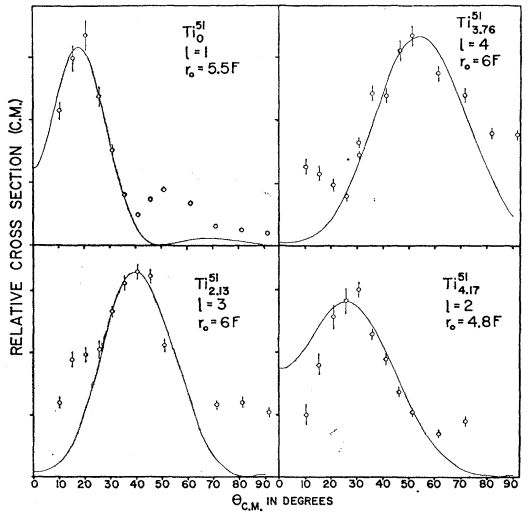

FIG. 2. Angular distributions observed in the $Ti^{50}(d,p)Ti^{51}$ reaction leading to the 0 ($l=1$), 2.13 ($l=3$), 3.76 ($l=4$), and 4.17 ($l=2$)-MeV states in Ti^{51}. Solid lines are Butler fits to the experimental points.

III. THEORETICAL INTERPRETATION

The spectrum of Ca^{49} is single-particle-like and has two levels, the ground and first excited states with $l_n=1$. On the basis of their intensity ratios and l_n values, these two levels have been assigned to the shell model $p_{3/2}$ and $p_{1/2}$ states.[5] In Ti^{51}, at least four levels (0-, 1.16-, 2.18-, and 2.89-MeV excitation) have $l_n=1$ and appear to share the $p_{3/2}$ and $p_{1/2}$ strength. A similar situation occurs in the case of Cr^{53} and Fe^{55}.

Even-even nuclei in the mass 50 region have a 2+ first excited state. In those even-even nuclei for which measurements have been made, the 2+ first excited state in its decay to the ground state shows a collective enhancement.[13,14] Raz[15,16] has obtained good agreement with experimental information regarding energies, spins, etc., of levels of even-even nuclei with two nucleons in the $f_{7/2}$ shell in a calculation which takes into account collective and interparticle interactions.

Therefore, it was considered reasonable to treat the core excitations in the present case as being collective

TABLE I. Summary of results obtained from $Ti^{50}(d,p)Ti^{51}$ stripping analysis.

Level	Excitation energy (MeV)	l_n	r_0 (fermis)	$(2J+1)\theta^2$ (rel.)
0	0	1	5.5	4
1	1.16	1	4.5	1.6
2	1.42	(3)	6.0	0.5
3	1.55	...	...	...
4	2.13	3	6.0	1.3
5	2.18	1	4.5	0.3
6	2.89	1	4.0	0.9
7	3.17	...	...	...
8	3.76	4	6.0	4.3
9	4.17	2	4.8	1.0
10	4.56	...	...	...
11	4.60	2	4.5	1.9

[13] K. Alder, A. Bohr, B. R. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).
[14] Y. Kh. Lemberg, in *Reaction between Complex Nuclei*, edited by A. Zucker *et al.* (John Wiley & Sons, Inc., New York, 1960).
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in nature, arising out of quadrupole oscillations of the even-even core. The lowest shell model states for the odd neutron—in this case the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$—were coupled to the phonon states of the core. From the information available for other nuclei in this mass region,[13,16] it appeared reasonable to assume that the surface to particle coupling in the present case was intermediate rather than the weak (nuclei near double closed shells) or strong (deformed nuclei) coupling situations. An analogous calculation has been made in the case of the Cu isotopes by Bouten and Van Leuven.[17]

Intermediate coupling calculations involving diagonalization of the interaction Hamiltonian including several phonons of excitation have been dealt with by several authors.[15,18,19] The nuclear Hamiltonian can be written as

$$H=H_p+H_c+H_{\rm int},$$

where H_p and H_c are the particle and core Hamiltonians, respectively. $H_{\rm int}$ is given by

$$H_{\rm int}=-r\frac{\partial V}{\partial r}\left(\frac{\hbar\omega}{2C}\right)^{1/2}\sum_{\mu}[b_\mu+(-1)^\mu b_\mu^*]Y_{2\mu},$$

where $\hbar\omega$ is the phonon energy and C the surface deformation parameter; b_μ^* and b_μ are creation and destruction operators for phonons of spin 2 with Z component μ. $Y_{2\mu}$ is a normalized spherical harmonic of the angular coordinates of the particle. V is the shell-model potential for the particle and r its radial coordinate.

The wave functions can be written as

$$\Psi_{JM_J}=\sum_{NRj}a_{NRj}|NRj;JM_J\rangle,$$

where N is the number of phonons; R the spin of the core state is coupled to the particle state j to give a total spin J. To a good approximation the wavefunction can be separated into a particle part and a collective part. In such a representation H_c and H_p contribute diagonal terms, viz., the phonon energy and the single particle energies. The cores of the three nuclei being considered here need not necessarily have properties identical with those of Ti^{50}, Cr^{52}, and Fe^{54}. Also one cannot determine experimentally the single particle $(p_{1/2}-p_{3/2})$ and $(f_{5/2}-p_{3/2})$ energies in these nuclei. So, these quantities, namely, the phonon energy $\hbar\omega$, and the single particle $(p_{1/2}-p_{3/2})$ and $(f_{5/2}-p_{3/2})$ energies have been treated as free parameters in the present calculation. Values of these parameters were chosen to give a good fit to the spectra and level properties of the three nuclei (see Table II and Fig. 5).

$H_{\rm int}$ contributes the off-diagonal matrix elements of the Hamiltonian for which the matrix elements of

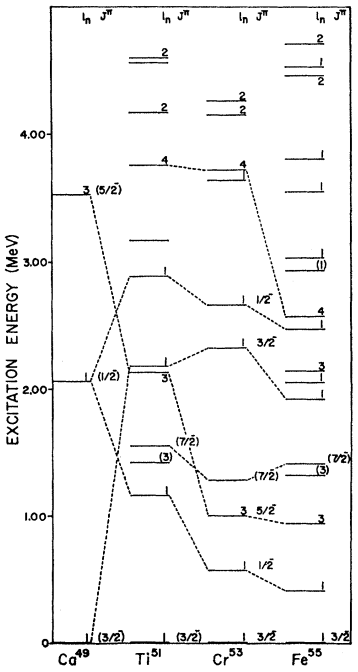

FIG. 3. Experimental spectra of Ca^{49}, Ti^{51}, Cr^{53}, and Fe^{55}.

$r(\partial V/\partial r)$ were treated as constants in the present calculation. The strength of the interaction is denoted by a dimensionless parameter x (see Table II); x is given by[20]

$$x=k(r)[5/(16\pi\hbar\omega C)]^{1/2},$$

where $k(r)=\langle|r(\partial V/\partial r)|\rangle$.

It was noticed that the lowest two eigenvalues for the three spin cases $(J=1/2, 3/2, 5/2)$ did not, in general, differ by more than 10% as between a two-phonon and a three-phonon calculation. Considering the limitations of the model, a three-phonon cutoff for the calculation was considered adequate. The resulting matrices, with maximum dimensions of 19×19 in the $J=5/2$ case were diagonalized by the University of Michigan IBM 7090 computer. Experimental and theoretical spectra for Ti^{51}, Cr^{53}, and Fe^{55} are plotted in Fig. 4. The relative spectroscopic factor, $(2J+1)S$ together with the excitation energies of the different levels obtained from theory and experiment are given in Table III. The relative $(2J+1)S$ listed in column (6) is obtained from the corresponding relative $(2J+1)\theta^2$

TABLE II. Parameters used in the calculations for Ti^{51}, Cr^{53}, and Fe^{55}. (Parameters are defined in text.)

	Ti^{51}	Cr^{53}	Fe^{55}
$(p_{1/2}-p_{3/2})$	2.1	2.1	1.2
$(f_{5/2}-p_{3/2})$	2.3	1.6	1.3
$\hbar\omega$	1.93	1.7	1.8
x	0.51	0.92	0.61

[17] M. Bouten and P. Van Leuven, Nucl. Phys. **32**, 499 (1962).
[18] D. C. Choudhri, Kgl. Danske Viedenskab. Selskab, Mat. Fys. Medd. **28**, Nr. 4 (1954).
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TABLE III. Excitations and relative spectroscopic factors obtained from experiment and theory for Ti^{51}, Cr^{53}, and Fe^{55}.

	Experimental						Theoretical		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Level	Exc. energy (MeV)	l_n	J^π	$(2J+1)\theta^2$ (rel.)	$(2J+1)S$ (rel.)	Exc. energy (MeV)	J^π	$(2J+1)S$ (rel.)
	0	0	1		4.0	4.0	0	$3/2^-$(1st)	4.0
	1	1.16	1		1.6	1.6	1.08	$1/2^-$(1st)	1.0
	2	1.42	(3)		0.5	0.9	1.67	$5/2^-$(1st)	1.7
Ti^{51}	3	1.55		$(7/2^-)$	0	0	1.78	$7/2^-$(1st)	0
	4	2.13	3		1.3	2.2	2.32	$5/2^-$(2nd)	2.7
	5	2.18	1		0.3	0.3	2.16	$3/2^-$(2nd)	0.4
	6	2.89	1		0.9	0.9	2.75	$1/2^-$(2nd)	0.9
	0	0	1	$3/2^-$	4.0	4.0	0	$3/2^-$(1st)	4.0
	1	0.57	1	$1/2^-$	1.5	1.5	0.39	$1/2^-$(1st)	1.0
	2	1.00	3	$5/2^-$	1.6	2.7	0.95	$5/2^-$(1st)	3.0
	3	1.28		$(7/2^-)$	0	0	1.39	$7/2^-$(1st)	0
	4						1.89	$5/2^-$(2nd)	0.3
Cr^{53} [a]	5						2.20	$3/2^-$(2nd)	0.1
	6	2.32	1	$3/2^-$	2.0	2.0	2.25	$3/2^-$(3rd)	1.6
	7						2.50	$5/2^-$(3rd)	2.2
	8	2.66	1	$(1/2^-)$	0.3	0.3	2.86	$1/2^-$(2nd)	0.6
	9						3.20	$5/2^-$(4th)	2.5
	10	3.65	1	$(3/2^-)$	1.6	1.6	3.45	$1/2^-$(3rd)	0.5
	0	0	1		4.0	4.0	0	$3/2^-$(1st)	4.0
	1	0.41	1		1.6	1.6	0.41	$1/2^-$(1st)	1.3
	2	0.94	3		2.6	4.4	0.99	$5/2^-$(1st)	4.2
	3	1.32	(3)		0.5	0.9	1.93	$5/2^-$(2nd)	0.4
Fe^{55} [b]	4	1.41		$(7/2^-)$	0	0	1.59	$7/2^-$(1st)	0
	5	1.92	1		0.1	0.1	2.09	$3/2^-$(2nd)	0.6
	6	2.05	1		0.3	0.3	2.25	$3/2^-$(3rd)	0.3
	7	2.14	3		1.0	1.7	2.37	$5/2^-$(3rd)	1.6
	8	2.47	1		0.9	0.9	2.43	$1/2^-$(2nd)	0.6
	9	3.03	1		0.1	0.1	3.17	$1/2^-$(3rd)	0.3

[a] Experimental results are from Refs. 1, 3, 23, and 24. [b] Experimental results are from Ref. 4.

listed in column (5) by setting $\theta_0^2(2p_{3/2})/\theta_0^2(2p_{1/2})=1$ and $\theta_0^2(2p_{3/2})/\theta_0^2(1f_{5/2})=1.7$ following the analysis of Macfarlane and French[21] in regard to empirical stripping reduced widths.

Ti^{51}

The ground-state wave function for Ti^{51} is found to be mostly the $NRj=(003/2)$ configuration. The model predicts a $J=3/2$ level at 2.16 MeV with a transition strength of 10% of the ground state, corresponding to the experimentally observed level at 2.18 MeV with $l_n=1$. The $J=1/2$ states are thoroughly mixed; two levels at 1.08 and 2.75 MeV share the $p_{1/2}$ strength and correspond to the experimentally observed levels at 1.16 and 2.89 MeV both of which have $l=1$. Thus the calculated energies and transition strengths of the four levels at 0, 1.08, 2.16, and 2.75 MeV are in qualitative agreement with experiment. The spin predictions remain to be verified by experiment. The first $J=5/2$ level predicted by the model is at 1.67 MeV. As stated earlier, the l_n values of two levels at 1.42 and 1.55 MeV could not be determined. Recently, results[22] have become available of a low-resolution (d,p) experiment performed on a Ti^{50} target at an incident deuteron energy of 21.4 MeV. At this deuteron energy, the intensity of $l_n=3$ transitions relative to $l_n=1$ transitions is higher than at lower energies. This has enabled Yntema to assign an $l_n=3$ to the level at 1.42 MeV, shown in parentheses in Table I. A second calculated $J=5/2$ level at 2.32 MeV corresponds to the 2.13-MeV level with an $l_n=3$, observed in the present experiment. The $f_{7/2}$ shell is filled for the neutrons and enables a fairly unambigous assignment of a 5/2 spin to be made to the low-lying levels of Ti^{51} which display an angular distribution of $l_n=3$.

Of the six low-lying levels in Ti^{51}, only one level at 1.55 MeV has not been discussed. As has been stated earlier the angular distribution of this level over an angular range of 25° to 45° was found to be isotropic and its intensity low. This is assigned a spin 7/2, a state which can arise out of the coupling of the one phonon 2+ core state to the lowest neutron state—the $p_{3/2}$. A level with $J=7/2$ is not expected to be seen in a stripping experiment as the $f_{7/2}$ shell is filled for the neutrons and its stripping width is zero. Three-phonon calculations have been made to fit the energy eigenvalue and the result is a 7/2 level at 1.78 MeV.

Positive parity states start above 3.0 MeV of excitation. Thus, a level at 3.76 MeV is probably a major component of the $g_{9/2}$ single particle state. Levels at

[21] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).
[22] J. L. Yntema (unpublished).

4.17 and 4.60 MeV arise out of neutron excitations into the $2d$ shell.

Cr^{53}

Extensive experimental information is available in the case of this nucleus. Apart from information derived from the (d,p) reaction on Cr^{52}, the spins of individual levels of Cr^{53} have been measured by Van Patter *et al.*[23] and by Bartholomew and Gunye.[24] The available experimental information in regard to the level properties of Cr^{53} are found in Table III, an interesting feature of which is the relatively large reduced width for the spin 3/2 level at 2.32 MeV. The main attempt in this case was to vary the parameters of the calculation to match the spin, energy, and reduced width of this level. This necessitated the use of a larger value for the coupling constant ($x=0.92$ for Cr^{53} as against 0.51 for Ti^{51} and 0.61 for Fe^{55}), an interpretation of which will be discussed in Sec. IV. The calculation yielded a spin 3/2 level at 2.25 MeV having 4/10th of the reduced width for the ground-state transition. For the spin 1/2 case, this value of the coupling constant produced a somewhat larger repulsion for the lowest two eigenvalues giving two levels at 0.39- and 2.86-MeV excitation having 52% of the $p_{1/2}$ strength. The experimentally observed $J=1/2$ levels are at 0.57 and 2.66 MeV. An additional spin 1/2 level at 3.45 MeV is obtained from the calculations having 18% of the $p_{1/2}$ strength. The calculation therefore favors a spin assignment of 1/2 for the experimentally observed level at 3.65 MeV with $l_n=1$ as nearly all of the calculated $p_{3/2}$ strength was taken up by the states at 0, 2.20, and 2.25 MeV (Table III). The model predicts a $J=5/2$ level at 0.95 MeV with a reduced width comparable with the experimentally observed one at 1.00 MeV, having an $l_n=3$. Two more levels with $J=5/2$ and of comparable intensity are predicted at 2.50 and 3.20 MeV, but corresponding levels have not been seen experimentally. A level with spin 7/2 is obtained from the calculations at 1.39 MeV. This compares with an experimentally observed level at 1.28 MeV for which a spin assignment of 7/2 is consistent with the results of Van Patter *et al.*[23]

Moments and Transitions for Cr^{53}

The eigenfunction obtained for the ground state of Cr^{53} has been used to compute the magnetic moment. The magnetic moment is given by

$$\langle\Psi|\mu|\Psi\rangle=\sum_{N'R'j'}\sum_{NRj}a^*_{N'R'j'}a_{NRj}\langle N'R'j'JM_J=J|\mu|NRjJM_J=J\rangle.$$

We take $\boldsymbol{\mu}_{\rm op}$ as

$$\boldsymbol{\mu}_{\rm op}=g_R\mathbf{J}+(g_s-g_R)\mathbf{j}+(g_l-g_s)\mathbf{l}$$

where g_R is the g factor for the nuclear core and is assumed to be equal to Z/A. The other symbols have their usual significance. We have

$$\langle NRj'JM_J|\mu_Z|NRjJM_J\rangle=g_RM_J+\frac{M_J}{[J(J+1)(2J+1)]^{1/2}}(-1)^{j+1+R+J}(2J+1)\times\begin{Bmatrix}j & j' & 1\\ J & J & R\end{Bmatrix}\Big\{[j(j+1)(2j+1)]^{1/2}(g_s-g_R)\delta_{jj'}+[(2j+1)(2j'+1)]^{1/2}(-1)^{l'+1+\frac{1}{2}+j}\begin{Bmatrix}l & l' & 1\\ j' & j & \frac{1}{2}\end{Bmatrix}\times[l(l+1)(2l+1)]^{1/2}(g_l-g_s)\delta_{ll'}\Big\}.$$

It is seen that in this case terms with $j\neq j'$ also contribute to the calculated magnetic moment. The magnetic moment thus obtained is -1.56 nm; the experimental value is -0.47 nm while the Schmidt value is -1.9 nm. The magnetic moment computed from the ground-state wavefunction is seen to be closer to the Schmidt value. It appears that the model fails to describe the ground-state properties of Cr^{53}; a similar disagreement between the calculated and experimental values is noticed for the magnetic moments of Cu^{63} and Cu^{65}.[17]

The static quadrupole moment was computed using the ground-state wavefunction of Cr^{53} and the result was

$$\langle|Q|\rangle=-2.07[\hbar\omega/(2C)]^{1/2}b.$$

Comparing this with the experimental value[25] for the quadrupole moment of Cr^{53} of $-0.03b$, one obtains an unreasonably large value for the surface deformation parameter C of about 3800 MeV. This value for the quadrupole moment has been deduced from the hyperfine spectrum of Cr^{53} in Al_2O_3 and was based on certain assumptions about the relative values of the electric-field gradient at the two ion sites. More recent information[26] for Cr^{53} and V^{51} in a corundum lattice indicates appreciable distortion of this lattice by Cr^{53}. A similar distortion in the case of Cr^{53} in Al_2O_3 would make the assumption above less valid and the magnitude of the quadrupole moment less certain.

The lifetime of the 2.32-MeV state of Cr^{53} has been reported as $(4\pm2)10^{-15}$ sec.[27,28] Expressed as transition probabilities, $B(E2)$ for this state is approximately equal to 200 times the single-particle estimate, while $B(M1)$ is approximately equal to 1/5 the single-particle estimate. It was assumed that the transition is predominantly $M1$. Using the wavefunctions obtained for

[23] D. M. Van Patter, N. Nath, S. M. Sharfroth, S. S. Malik, and M. A. Rothman, Phys. Rev. **128**, 1246 (1962).
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[25] R. W. Terhune, J. Lambe, C. KiKuchi, and J. Baker, Phys. Rev. **123**, 1265 (1961).
[26] C. KiKuchi (private communication).
[27] E. C. Booth and K. A. Wright, Bull. Am. Phys. Soc. **8**, 85 (1963).
[28] E. C. Booth (private communication).

the ground state and 2.32-MeV state of Cr^{53}, the transition probability for the 2.32-MeV level was computed and the following result was obtained:

$$B(M1)=0.08\,\mu_N^2 \quad \text{calculated},$$
$$B(M1)=1.16\,\mu_N^2 \quad \text{experimental}.$$

The calculated value is too small compared to the experimental by a factor of about 15.

Fe^{55}

To fit the experimental data of Fe^{55} up to an excitation of 3 MeV, it was found necessary to lower the $(p_{1/2}-p_{3/2})$ single particle energy from 2.1 (for Ti^{51} and Cr^{53}) to 1.2 MeV. The $(f_{5/2}-p_{3/2})$ single particle energy shows a smooth variation from Ti^{51} to Fe^{55} (Fig. 5). The spins of the various levels of Fe^{55} are not known. For the levels having $l_n=1$ as observed in a (d,p) reaction, there is an ambiguity in spins. Calculation in this case gives two $J=3/2$ levels at 2.09 and 2.25 MeV with about a sixth of the ground-state transition strength, corresponding to the observed levels at 1.92 and 2.05 with $l_n=1$. The agreement for the energies and transition strengths of the $J=5/2$ and $J=1/2$ levels is reasonable (Table III), provided the spin assignments (Fig. 4) are correct. The calculated 7/2 level is at 1.59 MeV and can be compared with a level at 1.41 MeV with an isotropic angular distribution as seen in a (d,p) reaction, which appears to be the best candidate for a spin 7/2 assignment.

IV. CONCLUSION

The single particle energies of the $p_{1/2}$ and $f_{5/2}$ states relative to the $p_{3/2}$ state, the phonon energies and values of the parameter "x" used in the present calculations are collected in Table II. It is noticed that the $f_{5/2}$ energy decreases smoothly (Fig. 5) with increasing mass number. The $p_{1/2}$ energy remains constant between Ca^{49} and Cr^{53} but shows an abrupt decrease for Fe^{55}. The quasiparticle $p_{1/2}$ and $f_{5/2}$ energies of the present calculation are plotted in Fig. 5 together with those obtained for the Cu isotopes[17] for purposes of comparison.

The frequency ω of the harmonic oscillator (the even-even core) is related to the surface deformation parameter, C and the inertial parameter, B through the relation

$$\omega=(C/B)^{1/2}.$$

H_{int} is proportional to $[\hbar\omega/(2C)]^{1/2}$, i.e., to the 4th root of the product (BC). If one makes the assumption that the inertial parameter B has a similar value in the three nuclei, the fact that the value of the coupling constant used in Cr^{53} is larger than in the other two cases would imply a smaller value for the surface deformation parameter C for Cr^{53}. This is also consistent with the fact that the phonon energy $\hbar\omega$ used in the calculation is smaller for Cr^{53}. The core of Cr^{53} which has four protons outside the closed $(1d-2s)$ proton shell is seen to be "softer" than the cores of Ti^{51} and Fe^{55} which have two particles or two holes in the $f_{7/2}$ proton shell. This would imply that among the three nuclei, collective effects are strongest in Cr^{53}.

The model calculation reproduces the qualitative features of the spectra and level properties of the three nuclei, and suggests a method of explaining the fragmentation of the single-particle strengths. The model is less successful in predicting the ground-state magnetic moment of Cr^{53} and the transition probability for the 2.32-MeV state. There are indications that a shell-

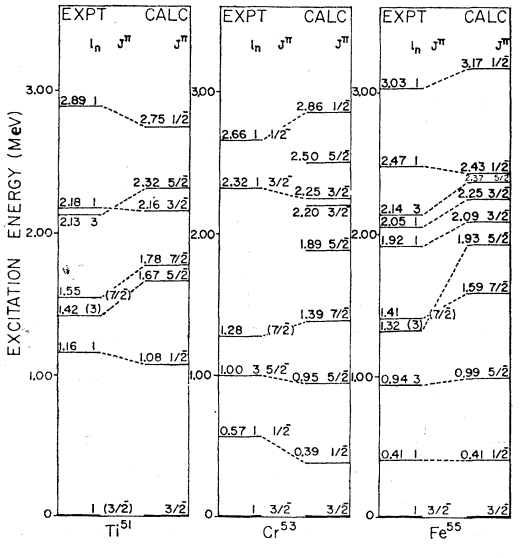

FIG. 4. Calculated and experimental spectra of Ti^{51}, Cr^{53}, and Fe^{55}.

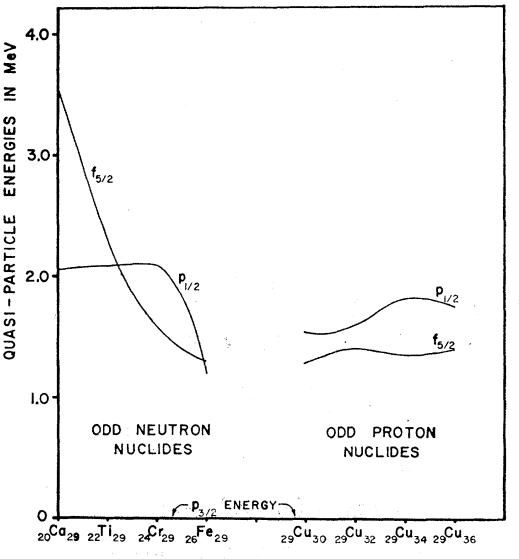

FIG. 5. Quasiparticle $p_{1/2}$ and $f_{5/2}$ energies used in the calculation for Ti^{51}, Cr^{53}, and Fe^{55}, together with those used in the calculation for Cu isotopes (Ref. 17).

model calculation[29] with a residual two-body interaction between the protons in the $f_{7/2}$ shell and the odd neutron is also successful in reproducing the features of the spectra of the three nuclei. A preliminary calculation using the shell-model wavefunction has given a value for the magnetic moment of Cr^{53} which is in better agreement with experiment than that obtained from the present calculations. A detailed comparison between the two models as applied to these nuclei should throw light on the limitations of the two models.

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Optical Model in the Interior of the Nucleus. II*

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Factors which influence the relative contribution of the interior and the surface of the nucleus to matrix elements for direct interactions involving nucleons in the entrance and exit channels are studied quantitatively. Purely optical-model effects causing localization of the reaction are phase averaging, which tends to de-emphasize the interior at all energies, and focusing which emphasizes the interior at low energies and the surface at higher energies. It is shown that phase averaging does not make the central contribution negligible at any energy. The foci in the optical-model wave functions have large effects on angular distributions. Density dependence of the two-body force for reactions which proceed by a two-body collision mechanism can be identified from angular distributions and from the energy dependence of backward cross sections which are particularly sensitive to the foci.

1. INTRODUCTION

IN a previous publication[1] (referred to as I) the question was discussed whether it is possible to infer anything about the radial localization of a direct interaction involving nucleons in the initial and final states from the general shape of the angular distributions.

The surface interaction model[2] for the excitation of collective states has had considerable success in predicting experimental results. The validity of the model is discussed particularly by Buck.[2] Direct interactions which proceed by a two-body collision in the nucleus have often been regarded also as surface effects for two main reasons.

The first concerns the optical-model wave functions which are used to represent initial and final states in the distorted-wave Born approximation (DWBA). Simple considerations[3] seem to indicate that the product particle would be likely to be reflected back into the nucleus if it came from the interior region. It was shown in I how a reduction of the interior contribution to the matrix element could arise from the fact that the phase of each partial wave of low-angular momentum is a smoother function of r in a distorted wave than in a plane wave. This effect has been called[4] "phase averaging." It is discussed for α particles by Rost.[4]

The second possible reason for reduction of the interior contribution to the matrix element is that it might be due to the reaction mechanism. For example, the fact that the Pauli principle is expected to inhibit two-body reactions more in dense nuclear matter than in the surface leads to a surface localization. There is evidence from doublet splitting that effective two-body forces in the shell model are density dependent.[5]

It was shown in I that, for low-energy direct interactions, a qualitative difference is to be expected between angular distribution shapes for surface and

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